ME 223
FLUID MECHANICS & MACHINERY

FLUID STATICS

LECTURE 3

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LINEARLY ACCELERATING CONTAINERS

In this section the fluid will be at rest relative to a reference frame that is linearly accelerating with a horizontal component a_x and a vertical component a_z. Then the total pressure is simplified as

$$dp = -\rho\, a_x\, dx\; - \rho\,(\,a_z + g)\, dz$$

Integrating between two arbitrary points 1 and 2 results in

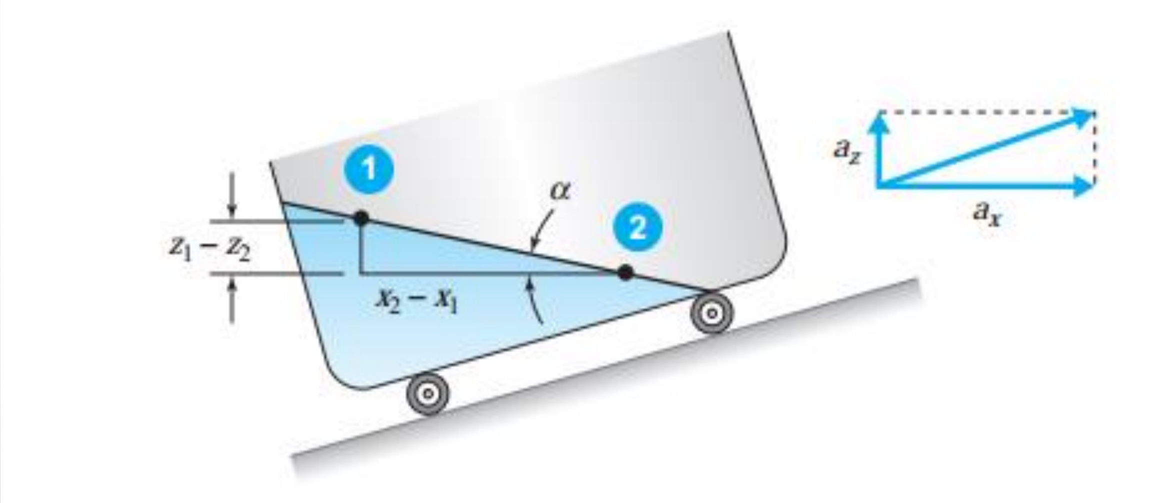


Fig. Linearly Accelerating containers

LINEARLY ACCELERATING CONTAINERS

$$p_2 - p_1 = -\rho a_x(x_2 - x_1) - \rho(a_z + g)(z_2 - z_1)$$

If points 1 and 2 lie on a constant-pressure line, such as the free surface in the above figure, then $p_2 - p_1 = 0$ and we have

$$\frac{z_1 - z_2}{x_2 - x_1} = tan\alpha = \frac{a_x}{a_z + g}$$

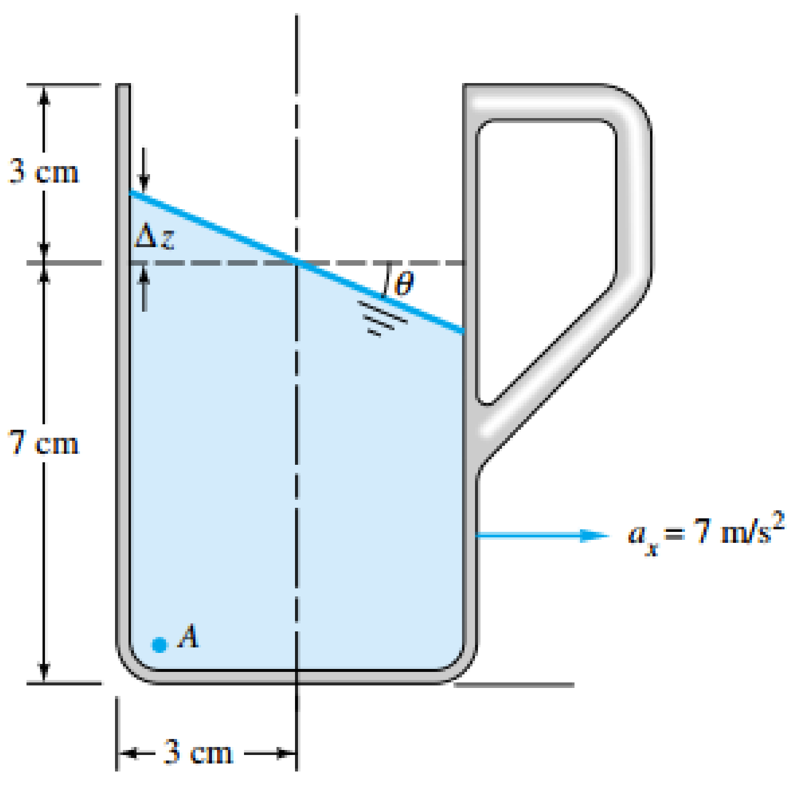
where α is the angle that the constant-pressure line makes with the horizontal.

Note that density or viscosity does not appear in the above equation.

LINEARLY ACCELERATING CONTAINERS

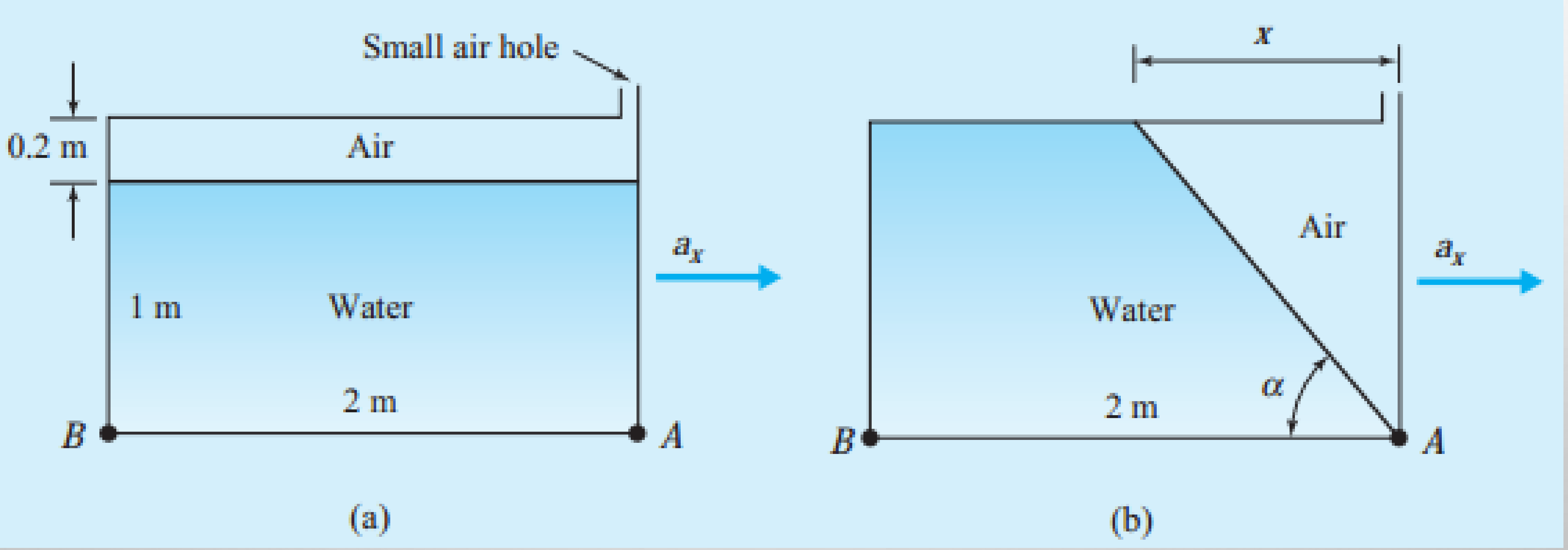
PROBLEM 1:

A drag racer rests her coffee mug on a horizontal tray while she accelerates at $7\ m/s^2$. The mug is 10 cm deep and 6 cm in diameter and contains coffee 7 cm deep at rest. (a) Assuming rigid body acceleration of the coffee, determine whether it will spill out of the mug. (b) Calculate the gage pressure in the corner at point A if the density of coffee is $1010\ \frac{kg}{m^3}$.



LINEARLY ACCELERATING CONTAINERS

PROBLEM 2:



LINEARLY ACCELERATING CONTAINERS

The tank shown in Fig. is accelerated to the right. Calculate the acceleration a_x needed to cause the free surface, shown in Fig., to touch point A. Also, find p_B and the total force acting on the bottom of the tank if the tank width is 1 m.

ROTATING CONTAINERS

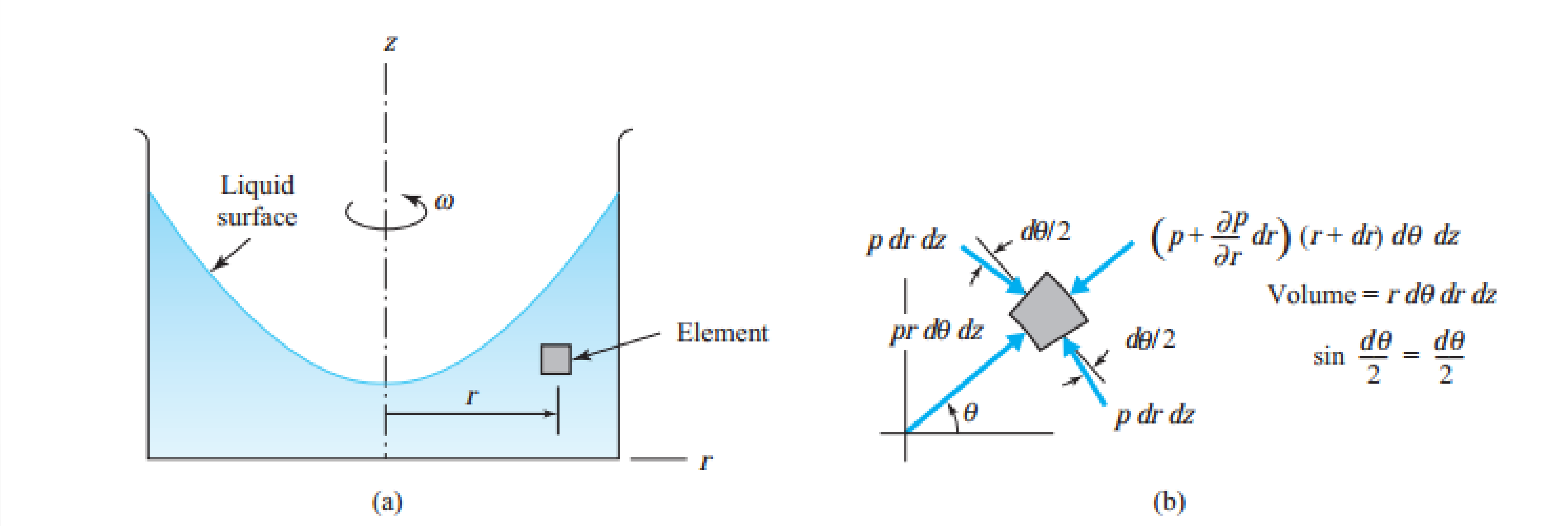


Fig. Rotating Container: a) Liquid cross-section, b) top view of element

ROTATING CONTAINERS

In this section we consider the situation of a liquid contained in a rotating container, such as that shown in Fig. before. After a relatively short time the liquid reaches static equilibrium with respect to the container and the rotating rz-reference frame. There will be no variation of pressure with respect to the θ-coordinate. Applying Newton's second law ($\sum F_r = ma_r$) in the r-direction to the element shown,

ROTATING CONTAINERS

$$p r d\theta dz - \left(p + \frac{\partial p}{\partial r} dr\right)(r + dr) d\theta dz + p dr dz \sin\frac{d\theta}{2} + p dr dz \sin\frac{d\theta}{2}$$
$$= -(\rho r d\theta dr dz)\,\omega^2 r$$

$$p r d\theta dz - p r d\theta dz - p dr d\theta dz - \frac{\partial p}{\partial r} dr\, r d\theta dz - \frac{\partial p}{\partial r}\,(\cancel{dr})^2 d\theta dz + 2 p dr dz \frac{d\theta}{2}$$
$$= -(\rho r d\theta dr dz)\,\omega^2 r$$

where the acceleration is $\omega^2 r$ toward the center of rotation. Simplify and divide by the volume $r d\theta\, dr dz$; then

ROTATING CONTAINERS

$$\frac{\partial p}{\partial r} = \rho r \omega^2$$

Here we have neglected the higher-order term that contains the differential dr.

The pressure differential then becomes

$$dp = \frac{\partial p}{\partial r} dr + \frac{\partial p}{\partial z} dz$$
$$= \rho r \omega^2 \, dr - \rho g dz$$

where we have used the static pressure variation given by the above Eq. with

$a_z = 0$. We can now integrate between any two points (r_1, z_1) and (r_2, z_2)

ROTATING CONTAINERS

$$p_2 - p_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1)$$

If the two points are on a constant-pressure surface, such as the free surface, locating point 1 on the z-axis so that $r_1 = 0$, there results

$$\frac{\omega^2 r_2^2}{2} = g(z_2 - z_1)$$

which is the equation of a parabola. Hence the free surface is a paraboloid of revolution. The equations above can now, with the conservation of mass, be used to solve problems of interest.

Note that density or viscosity does not appear in the above equation.

ROTATING CONTAINERS

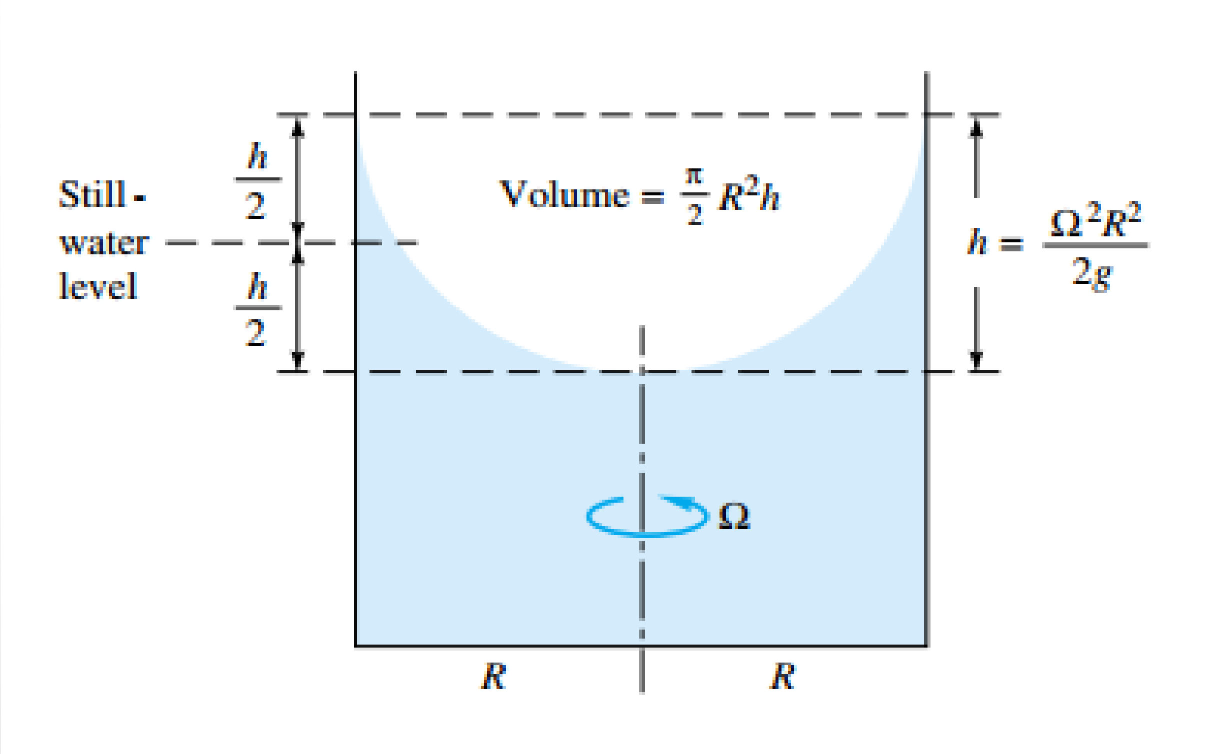


Fig. Determining the free surface position for rotation of a cylinder of fluid about its central axis.

ROTATING CONTAINERS

If $r_2 = R =$ the container radius, then the total rise of water level at the wall,

$$h = z_2 - z_1 = \frac{\omega^2 R^2}{2g}$$

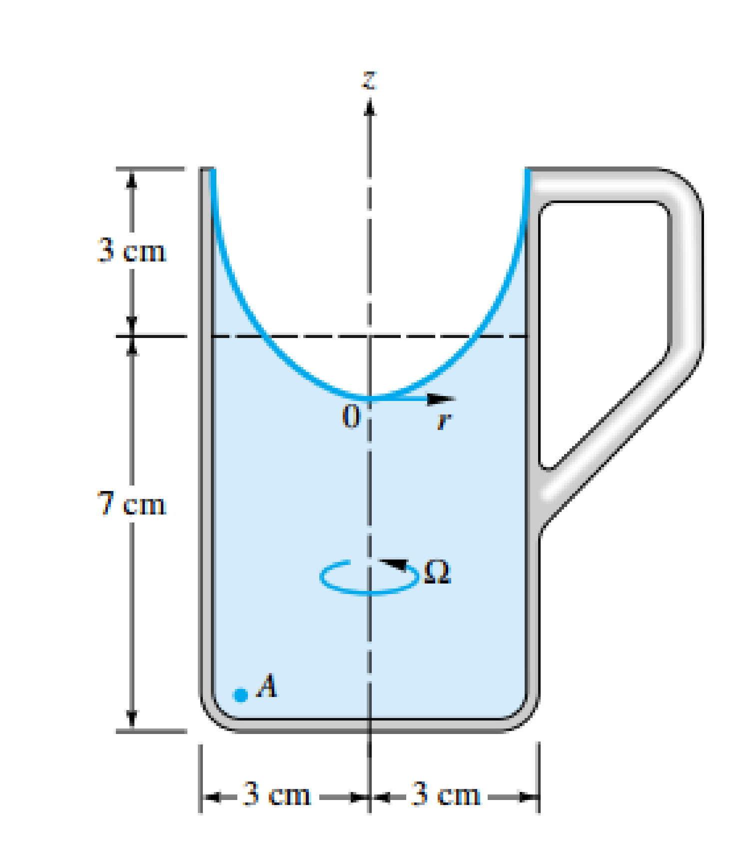
Since the volume of a paraboloid is one-half the base area time height, the still – water level is exactly halfway between the high and low points of the free surface.

That is the center of the fluid drops an amount, $\frac{h}{2} = \frac{\omega^2 R^2}{4g}$ and the edges rise an equal amount.

ROTATING CONTAINERS

PROBLEM 3:

The coffee cup is removed from the drag racer, placed on a turntable, and rotated about its central axis until a rigid-body mode occurs. Find (a) the angular velocity which will cause the coffee to just reach the lip of the cup and (b) the gage pressure at point A for this condition.



ROTATING CONTAINERS

PROBLEM 4:

The cylinder shown in Fig. is rotated about its centerline. Calculate the rotational speed that is necessary for the water to just touch the origin O. Also, find the pressures at A and B.